Mark Scheme (Results)

Summer 2018

Pearson Edexcel International A Level
In Further Pure Mathmatics F3
(WFM03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

June 2018
WFM03 Further Pure Mathematics F3
Mark Scheme

Question Number	Scheme	Notes	Marks
1	$15\text{sech}^2 x + 7\tanh x = 13$		
	$15(1-\tanh^2 x) + 7\tanh x = 13$	Uses $\text{sech}^2 x = 1 - \tanh^2 x$	M1
	$15\tanh^2 x - 7\tanh x - 2 = 0$	Correct 3 term quadratic, terms in any order	A1
	$(5\tanh x + 1)(3\tanh x - 2) = 0$ $\Rightarrow \tanh x = -\frac{1}{5}, \frac{2}{3}$	M1: Solves their 3 term quadratic to obtain at least one value for tanh$x$ Correct answers implies method A1: Both correct values If solved by formula accept $\frac{7 \pm 13}{30}$	M1A1
	$x = \frac{1}{2}\ln\frac{2}{3}, \ \frac{1}{2}\ln 5$	A1: One correct exact answer A1: Both exact answers correct Allow equivalent answers e.g. $x = \frac{1}{2}\ln 2 - \frac{1}{2}\ln 3, \ln\frac{\sqrt{6}}{3}, \ln\sqrt{\frac{2}{3}}, \ln\sqrt{5}$ etc	A1, A1
			(6)
			Total 6
	Alternative Using Exponentials		
	$15\left(\frac{2}{e^x + e^{-x}}\right)^2 + 7\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = 13$	Substitutes the correct exponential forms The equation may have been re-arranged before substitution. ½s may have been cancelled.	M1
	$6e^{2x} - 34 + 20e^{-2x} = 0$	Correct 3 term quadratic in e^{2x}	A1
	$3e^{4x} - 17e^{2x} + 10 = 0$		
	$(3e^{2x} - 2)(e^{2x} - 5) = 0$ or $(3e^x - 2e^{-x})(e^x - 5e^{-x}) = 0$ $\Rightarrow e^{2x} = \frac{2}{3}$ or 5	M1: Solves their 3 term quadratic to obtain at least one value for e^{2x} A1: Both correct values	M1A1
	$x = \frac{1}{2}\ln\frac{2}{3}, \ \frac{1}{2}\ln 5$	A1: One correct answer A1: Both answers correct Allow equivalent answers e.g. $x = \frac{1}{2}\ln 2 - \frac{1}{2}\ln 3$	A1, A1

Solving quadratics by calculator: check their solutions if the equation is incorrect. If the solution is correct for their equation, award M1

Question Number	Scheme	Notes	Marks
2	$\mathbf{A}=\begin{pmatrix}3 & 2\\ 2 & 6\end{pmatrix}$		
(a)	$\det(\mathbf{A}-\lambda\mathbf{I})=0$ or $\begin{vmatrix}3-\lambda & 2\\ 2 & 6-\lambda\end{vmatrix}\ (=0)$	Forms the characteristic equation. = 0 may be missing	M1
	$(3-\lambda)(6-\lambda)-4(=0)$	Expands the determinant and attempts to solve the equation	M1
	$\lambda=2,7$	Correct eigenvalues obtained	A1
	$\begin{pmatrix}3 & 2\\ 2 & 6\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=2\begin{pmatrix}x\\ y\end{pmatrix}$ or $\begin{pmatrix}3 & 2\\ 2 & 6\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=7\begin{pmatrix}x\\ y\end{pmatrix}$	Use either of *their* eigenvalues to obtain at least one pair of non-zero values.	M1
	$\begin{pmatrix}3-2 & 2\\ 2 & 6-2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=0$ OR $\begin{pmatrix}3-7 & 2\\ 2 & 6-7\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=0$	Alt for line above	
	$\begin{pmatrix}1\\ 2\end{pmatrix},\begin{pmatrix}2\\ -1\end{pmatrix}$ or $x=1, y=2\ /\ x=2, y=-1$	A1: One correct pair of values (allow any multiples) A1: Both correct pairs of values (allow any multiples)	A1A1
	$\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{2}{\sqrt{5}}\end{pmatrix},\begin{pmatrix}\frac{2}{\sqrt{5}}\\ \frac{-1}{\sqrt{5}}\end{pmatrix}$ or $\frac{1}{\sqrt{5}}\begin{pmatrix}1\\ 2\end{pmatrix},\frac{1}{\sqrt{5}}\begin{pmatrix}2\\ -1\end{pmatrix}$	Both correct and normalised Follow through their eigenvectors	A1ft
			(7)
(b)	$\mathbf{D}=\begin{pmatrix}7 & 0\\ 0 & 2\end{pmatrix}$, $\mathbf{P}=\begin{pmatrix}\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}\\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}}\end{pmatrix}=\frac{1}{\sqrt{5}}\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$	B1ft: One correct ft (must be labelled) B1: Both fully correct and consistent (must both be labelled) (ie order of eigenvalues must be consistent with order of eigenvectors)	B1ft, B1
	$\mathbf{D}=\begin{pmatrix}0 & 7\\ 2 & 0\end{pmatrix}$, $\mathbf{P}=\begin{pmatrix}\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}\end{pmatrix}$	Both can be reversed and multiples allowed. $\mathbf{D}=k^2\times$matrix shown $\mathbf{P}=k\times$matrix shown	
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
3 **Way 1**	$\frac{\mathrm{d}\left(\frac{\sin x}{\cos x-1}\right)}{\mathrm{d}x}=\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}$	M1: Correct use of quotient (or product) rule A1: Correct expression	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\left(\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}\right)$	dM1: $\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\times$ quotient (or product) rule must be a function of x A1: Correct expression	dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\cos x-1)^2}{(\cos x-1)^2+\sin^2 x}\left(\frac{1-\cos x}{(\cos x-1)^2}\right)=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant A1: cao	ddM1A1
	Special Case: Quotient rule used with numerator terms wrong way round and work otherwise correct: award M1A0 and M1A0ddM1A0 if rest of method correct		**(6)** **Total 6**
Way 2	$\frac{\mathrm{d}\left(\frac{\sin x}{\cos x-1}\right)}{\mathrm{d}x}=\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}$	M1: Correct use of quotient (or product) rule A1: Correct expression	M1A1
	$\tan y=\left(\frac{\sin x}{\cos x-1}\right)\Rightarrow \sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\left(\frac{\cos x(\cos x-1)+\sin^2 x}{(\cos x-1)^2}\right)$	dM1: $\frac{1}{1+\left(\frac{\sin x}{\cos x-1}\right)^2}\times$ quotient (or product) rule must be a function of x A1: Correct expression	dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\cos x-1)^2}{(\cos x-1)^2+\sin^2 x}\left(\frac{1-\cos x}{(\cos x-1)^2}\right)=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant. A1: cao	ddM1A1
Way 3	$\tan y=\left(\frac{\sin x}{\cos x-1}\right)\Rightarrow(\cos x-1)\tan y=\sin x$		
	$\Rightarrow -\sin x\tan y+(\cos x-1)\sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=\cos x$	M1: Differentiates implicitly A1: Correct differentiation	M1A1
	$\Rightarrow\frac{-\sin^2 x}{\cos x-1}+(\cos x-1)\left(1+\frac{\sin^2 x}{(\cos x-1)^2}\right)\frac{\mathrm{d}y}{\mathrm{d}x}=\cos x$	dM1: Substitutes for y throughout A1: Correct equation in terms of x only (and $\mathrm{d}y/\mathrm{d}x$)	dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant. A1: cao	ddM1A1
Way 4	$\frac{\sin x}{\cos x-1}=\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1-2\sin^2\frac{x}{2}-1}$	M1: Using the correct double angle formula A1: Correct expression	M1A1
	$=-\cot\frac{x}{2}=-\tan\left(\frac{\pi}{2}\pm\frac{x}{2}\right)=\tan\left(\frac{x}{2}\pm\frac{\pi}{2}\right)$	M1: Obtains tan in terms of x A1: $\tan\left(\frac{x}{2}\pm\frac{\pi}{2}\right)$	dM1A1
	So $y=\arctan\left(\tan\left(\frac{x}{2}\pm\frac{\pi}{2}\right)\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$	ddM1: Attempts to simplify to obtain a constant. Must reach a constant. A1: cao	ddM1A1

Question Number	Scheme	Notes	Marks
4	$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$		
(a)	$\dfrac{dy}{dx}=\dfrac{b\sec^2\theta}{a\sec\theta\tan\theta}$ or $\dfrac{b^2x}{a^2y}$ or $\dfrac{bx}{a^2}\left(\dfrac{x^2}{a^2}-1\right)^{-\frac{1}{2}}$	Correct tangent gradient in any form	B1
	$m_N=-\dfrac{a\sec\theta\tan\theta}{b\sec^2\theta}\left(=-\dfrac{a}{b}\sin\theta\right)$	Use parametric forms and the correct perpendicular rule	M1
	$y-b\tan\theta=-\dfrac{a}{b}\sin\theta(x-a\sec\theta)$	M1: Correct straight line method using their m_N Use of $y=mx+c$ must include finding a value for c A1: Correct equation any equivalent to that shown.	M1A1
	$by-b^2\tan\theta=-ax\sin\theta+a^2\tan\theta$		
	$ax\sin\theta+by=(a^2+b^2)\tan\theta$ *	Completes to printed answer with at least one intermediate step	A1*
			(5)
(b)	$y=0\Rightarrow x=\dfrac{(a^2+b^2)\tan\theta}{a\sin\theta}\left(=\dfrac{(a^2+b^2)}{a}\sec\theta\right)$	Correct x coordinate	B1
	M is $\left(\dfrac{1}{2}\left(\dfrac{a^2+b^2}{a}\sec\theta+a\sec\theta\right),\dfrac{b}{2}\tan\theta\right)$ $=\left(\dfrac{2a^2+b^2}{2a}\sec\theta,\dfrac{b}{2}\tan\theta\right)$ oe	M1: Correct midpoint method for their x coordinate A1: Correct coordinates for M, any equivalent accepted. Need not be in coordinate brackets.	M1A1
			(3)
(c)	$\sec\theta=\dfrac{2ax}{2a^2+b^2},\tan\theta=\dfrac{2y}{b}\Rightarrow 1+\left(\dfrac{2y}{b}\right)^2=\left(\dfrac{2ax}{2a^2+b^2}\right)^2$	M1: Correct attempt to eliminate θ using coordinates of M A1: Correct equation	M1A1
	$y^2=\dfrac{b^2}{4}\left(\dfrac{4a^2x^2}{(2a^2+b^2)^2}-1\right)$ oe	dM1: Makes y^2 the subject A1: Correct equation in the required form	dM1A1
			(4)
			Total 12

Question Number	Scheme	Notes	Marks
5	$\mathbf{M}=\begin{pmatrix}4 & -5 & 0\\ k & 2 & 0\\ -3 & -5 & k\end{pmatrix}$		
(a)	$\lvert\mathbf{M}\rvert = 4(2k)+5(k^2)(+0)$	Correct determinant in any form (Quadratic may be unsimplified)	B1
	Minors : $\begin{pmatrix}2k & k^2 & -5k+6\\ -5k & 4k & -35\\ 0 & 0 & 8+5k\end{pmatrix}$ or cofactors : $\begin{pmatrix}2k & -k^2 & 6-5k\\ 5k & 4k & 35\\ 0 & 0 & 8+5k\end{pmatrix}$ B1: A correct first step of minors or cofactors		B1
	$\mathbf{M}^{-1}=\frac{1}{5k^2+8k}\begin{pmatrix}2k & 5k & 0\\ -k^2 & 4k & 0\\ 6-5k & 35 & 8+5k\end{pmatrix}$	M1: Fully recognisable attempt at the inverse including reciprocal of the determinant B1: Any 2 correct rows or columns ignoring determinant (may be missing) M mark not required A1: Fully correct inverse	M1B1A1
			(5)
(b)	$\mathbf{M}^{-1}=-\frac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix}$	Substitutes $k=-1$	M1
	$\Pi_2 : x=s, y=t, z=2s-4$	Attempts parametric form ($s\neq 0, t\neq 0$) Any pair of letters (inc x and y) can be used as parameters	M1
	$-\frac{1}{3}\begin{pmatrix}-2 & -5 & 0\\ -1 & -4 & 0\\ 11 & 35 & 3\end{pmatrix}\begin{pmatrix}s\\ t\\ 2s-4\end{pmatrix}$	Attempts $\mathbf{M}^{-1}\times$their parametric form Depends on both M marks above	ddM1
	$-\frac{1}{3}\begin{pmatrix}-2s-5t\\ -s-4t\\ 11s+35t+6s-12\end{pmatrix}$	Correct parametric form for Π_1 with s, t	A1
	$11x-5y+z=4$	dddM1:Eliminates s and t to obtain a cartesian equation All 3 previous M marks needed $x=-2x-5y$ gets M0 here (unless the parameters are now changed) A1:Correct equation (oe)	dddM1A1
			(6)
			Total 11

(b) Way 2	$\mathbf{M}=\begin{pmatrix}4 & -5 & 0\\ -1 & 2 & 0\\ -3 & -5 & -1\end{pmatrix}$		
	$\Pi_2 : x=s, y=t, z=2s-4$	Attempts parametric form	M1
	$\begin{pmatrix}4 & -5 & 0\\ -1 & 2 & 0\\ -3 & -5 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}4x-5y\\ -x+2y\\ -3x-5y-z\end{pmatrix}$	Attempts $\mathbf{M}\boldsymbol{x}$	M1
	$\begin{pmatrix}4x-5y\\ -x+2y\\ -3x-5y-z\end{pmatrix}=\begin{pmatrix}s\\ t\\ 2s-4\end{pmatrix}$	ddM1: Sets $\mathbf{M}\boldsymbol{x} =$ their parametric form A1: Correct equations	ddM1 A1
	$11x-5y+z=4$	M1:Eliminates s and t to obtain a cartesian equation A1:Correct equation (oe)	dddM1 A1

Way 3	$\begin{pmatrix}4 & -5 & 0\\ -1 & 2 & 0\\ -3 & -5 & -1\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$	M1: General point (a, b, c) on first plane M1: Setting up the transformation equation (as left)	M1 M1
	$4a-5b=x$ $-a+2b=y$ $-3a-5b-c=z$	M1: Multiply the matrices on the lhs and equate to rhs A1: correct equations	ddM1A1
	$2x-z=4 \Rightarrow 2(4a-5b)-(-3a-5b-c)=4$	M1: Using $2x-z=4$	dddM1
	$11a-5b+c=4$		
	$11x-5y+z=4$	A1: Correct equation of the plane. Must have x, y, z	A1

Question Number	Scheme	Notes	Marks
6	$x=\theta-\tanh\theta,\quad y=\operatorname{sech}\theta,\qquad 0\le\theta\le\ln 3$		
(a)(i)	$\left(\dfrac{dx}{d\theta}\right)=1-\operatorname{sech}^2\theta$	Correct derivative	B1
(ii)	$\left(\dfrac{dy}{d\theta}\right)=-\operatorname{sech}\theta\tanh\theta$ oe	Correct derivative	B1
	If both derivatives are in terms of a different variable but otherwise correct, allow B1B0. If one (or both) incorrect award B0B0		
			(2)
(b)	$S=(2\pi)\displaystyle\int \operatorname{sech}\theta\sqrt{\left(1-\operatorname{sech}^2\theta\right)^2+\left(-\operatorname{sech}\theta\tanh\theta\right)^2}\ (d\theta)$	Uses the correct formula with their derivatives 2π not needed	M1
	$S=2\pi\displaystyle\int \operatorname{sech}\theta\sqrt{1-\operatorname{sech}^2\theta}\ d\theta$		
	$S=2\pi\displaystyle\int \operatorname{sech}\theta\tanh\theta\ d\theta$	Correct integral after full simplification – 2π and limits not needed	A1
	$S=2\pi\left[-\operatorname{sech}\theta\right]$	Correct integration – limits not needed	A1
	$S=-2\pi\left(\operatorname{sech}(\ln 3)-\operatorname{sech}(0)\right)=0.8\pi$	dM1: Include 2π and use limits (0 to ln3) correctly in a multiple of $\operatorname{sech}\theta$ A1: cao and cso	dM1A1cao and cso
	Use of calculator: Correct integral, inc correct limits, shown followed by correct answer (multiple of π) scores full marks. No need to simplify the initial integral shown but if simplified incorrectly, only M mark can be awarded regardless of final answer. Incorrect answer given, mark as scheme.		
	Allow h (eg from tanh) to disappear as long as the functions are treated as hyperbolics.		
			(5)
			Total 7

Question Number	Scheme	Notes	Marks
7	Π_1: $x+y+z=3$, Π_2: $2x+3y-z=4$		
(a) Way 1	$x=\lambda \Rightarrow y=\frac{7}{4}-\frac{3}{4}\lambda$ or $\lambda=\frac{4y-7}{-3}$	M1: Obtains 2 equations connecting x, y or z with λ A1: Correct equations	M1A1
	$z=\frac{5}{4}-\frac{1}{4}\lambda$ or $\lambda=5-4z$	M1: Obtains 3 equations connecting x, y or z with λ A1: Correct equations	M1A1
	$\frac{x}{1}=\frac{7-4y}{3}=\frac{5-4z}{1}(=\lambda)$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
	$y=\lambda \Rightarrow \frac{7-3x}{4}=\frac{y}{1}=\frac{3z-2}{1}$ $\left(\text{or } \frac{7-3x}{4}=y=3z-2\right)$ $z=\lambda \Rightarrow \frac{5-x}{4}=\frac{y+2}{3}=\frac{z}{1}$ (or $=z$)		
			(6)
(a) Way 2	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix} = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$	M1: Attempt vector product of normals A1: Correct vector	M1A1
	$x=0 \Rightarrow y+z=3,\ 3y-z=4$ $\Rightarrow y=\frac{7}{4},\ z=\frac{5}{4} \to \left(0,\ \frac{7}{4},\ \frac{5}{4}\right)$ NB $y=0$ gives $x=\frac{7}{3},\ z=\frac{2}{3}$ $z=0$ gives $x=5,\ y=-2$	M1: Attempt a point on the line A1: Correct point (1, 1, 1) seen frequently	M1A1
	$\frac{x}{-4}=\frac{y-\frac{7}{4}}{3}=\frac{z-\frac{5}{4}}{1}(=\lambda)$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
	or $\frac{x-1}{-4}=\frac{y-1}{3}=\frac{z-1}{1}(=\lambda)$	Equation seen if (1, 1, 1) used	**(6)**
(a) Way 3	$x=-\frac{4}{3}y+\frac{7}{3}$	M1: Eliminates 1 variable A1: Correct equation	M1A1
	$x=5-4z$	M1: Eliminates 2nd variable A1: Correct equation	M1A1
	$\frac{x}{1}=-\frac{4}{3}y+\frac{7}{3}=5-4z$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
			(6)
(b)	$5(-4\lambda)-4\left(\frac{7}{4}+3\lambda\right)+4\left(\frac{5}{4}+\lambda\right)=12$	Substitutes parametric form of L into Π_3	M1
	$\lambda=-\frac{1}{2} \Rightarrow x=...,\ y=...,\ z=...$	Solves for λ and attempts coordinates	dM1
	$\left(2,\ \frac{1}{4},\ \frac{3}{4}\right)$ or $x=2,\ y=\frac{1}{4},\ z=\frac{3}{4}$ or $\begin{pmatrix} 2 \\ 1/4 \\ 3/4 \end{pmatrix}$	Correct coordinates	A1
			(3)

(b) Way 2	$5x-4.\frac{3}{4}\left(\frac{7}{3}-x\right)+4.\frac{1}{4}(5-x)=12$	Substitutes for y and z in terms of x into Π_3	M1
	$x=2 \Rightarrow y=...,z=...$	Solves for x and attempts other coordinates	dM1
	$\left(2,\frac{1}{4},\frac{3}{4}\right)$ or $x=2, y=\frac{1}{4}, z=\frac{3}{4}$ or $\begin{pmatrix}2\\1/4\\3/4\end{pmatrix}$	Correct coordinates	A1

(c)	$\begin{pmatrix}-2\\-\frac{1}{4}\\-\frac{3}{4}\end{pmatrix}\bullet\begin{pmatrix}-4\\3\\1\end{pmatrix}=\sqrt{\frac{37}{8}}\sqrt{26}\cos\theta$	Use scalar product between ± their $\overrightarrow{OA}$ and direction of their L	M1
	$\frac{13}{2}=\sqrt{\frac{37}{8}}\sqrt{26}\cos\theta \Rightarrow \theta=...$	Evaluate the scalar product and complete to $\theta=$ … (or the supplementary angle) (Check the product if the vectors are incorrect)	dM1
	$\theta=53.6^{\circ}$	cao	A1
			(3)
			Total 12

Question Number	Scheme	Notes	Marks
8	$I_n = \int \frac{x^n}{\sqrt{(x^2+k^2)}}\,\mathrm{d}x$		
(a)	$I_n = \int x^{n-1}x\left(x^2+k^2\right)^{-\frac{1}{2}}\,\mathrm{d}x$	Separates correctly (Without this there will be no progress.)	B1
	$I_n = x^{n-1}\left(x^2+k^2\right)^{\frac{1}{2}} - \int (n-1)x^{n-2}\left(x^2+k^2\right)^{\frac{1}{2}}\,\mathrm{d}x$	M1: Parts in the correct direction A1: Correct expression	M1A1
	$= \ldots - (n-1)\int \frac{x^{n-2}\left(x^2+k^2\right)}{\sqrt{(x^2+k^2)}}\,\mathrm{d}x$	Writes $\left(x^2+k^2\right)^{\frac{1}{2}}$ as $\frac{\left(x^2+k^2\right)}{\sqrt{(x^2+k^2)}}$	dM1
	$= \ldots - (n-1)\int \frac{x^n}{\sqrt{(x^2+k^2)}}\,\mathrm{d}x - (n-1)\int \frac{k^2x^{n-2}}{\sqrt{(x^2+k^2)}}\,\mathrm{d}x$	Correct separation	A1
	$I_n = x^{n-1}\left(x^2+k^2\right)^{\frac{1}{2}} - (n-1)I_n - (n-1)k^2I_{n-2}$	Introduces I_n and I_{n-2} on rhs depends on both M marks above	ddM1
	$I_n = \frac{x^{n-1}}{n}\left(x^2+k^2\right)^{\frac{1}{2}} - \frac{(n-1)}{n}k^2I_{n-2}$ *	Cso (Given answer!)	A1*
			(7)
(b)	$I_5 = \int \frac{x^5}{\sqrt{(x^2+1)}}\,\mathrm{d}x = \frac{x^4}{5}\left(x^2+1\right)^{\frac{1}{2}} - \frac{4}{5}I_3$	Correct first application of the reduction formula Can have k^2 instead of 1	M1
	$I_3 = \frac{x^2}{3}\left(x^2+1\right)^{\frac{1}{2}} - \frac{2}{3}I_1$	Correct second application of the reduction formula Can have k^2 instead of 1	M1
	$I_1 = \int \frac{x}{\sqrt{(x^2+1)}}\,\mathrm{d}x = \left[\sqrt{x^2+1}\right] \Rightarrow I_5 = \ldots$	$\int \frac{x}{\sqrt{(x^2+1)}}\,\mathrm{d}x = a\sqrt{x^2+1}$ And attempt I_5 using correct limits (k^2 or 1)	ddM1
	$\int_0^1 \frac{x^5}{\sqrt{(x^2+1)}}\,\mathrm{d}x = \frac{7}{15}\sqrt{2} - \frac{8}{15}$	A1: Either term correct A1: Both terms correct	A1A1 **(5)** **Total 12**
(b)			
Way 2	$I_1 = \int \frac{x}{\sqrt{(x^2+1)}}\,\mathrm{d}x = \sqrt{x^2+1}$	$\int \frac{x}{\sqrt{(x^2+1)}}\,\mathrm{d}x = a\sqrt{x^2+1}$ (k^2 or 1)	M1
	$I_3 = \frac{x^2}{3}\left(x^2+1\right)^{\frac{1}{2}} - \frac{2}{3}I_1$	Attempt I_3 by using the reduction formula (k^2 or 1)	M1
	$I_5 = \int \frac{x^5}{\sqrt{(x^2+1)}}\,\mathrm{d}x = \frac{x^4}{5}\left(x^2+1\right)^{\frac{1}{2}} - \frac{4}{5}I_3$ $= \frac{x^4}{5}\left(x^2+1\right)^{\frac{1}{2}} - \frac{4}{5}\left(\frac{x^2}{3}\left(x^2+1\right)^{\frac{1}{2}} - \frac{2}{3}\left(x^2+1\right)^{\frac{1}{2}}\right)$	Form a complete statement for I_5 and use the correct limits (k^2 or 1)	ddM1
	$\int_0^1 \frac{x^5}{\sqrt{(x^2+1)}}\,\mathrm{d}x = \frac{7}{15}\sqrt{2} - \frac{8}{15}$	A1: Either term correct A1: Both terms correct	A1A1

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